New developments for hydrological lumped models which preserve physically meaningful properties and parameters

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Abstract This paper aims at showing that, the physical properties of the basic hydrologic processes can only be retained in rainfall-runoff models at finer spatial scales, while, due to the inherent topological non-linearity, physically based lumped models can only be derived through an averaging process conditional upon a correct representation of additional phenomena, such as the soil filling and depletion hysteresis and the exfiltration at the end of a rainfall event, which can only be obtained via simulation using a distributed modelling approach. Starting from the point equations and their physically meaningful parameterisation, the present work follows three lumping procedures: (1) a “process and parameter lumping” from the point equation to the pixel scale equation in order to describe the phenomena in discrete space; (2) a “geo-morphological lumping” from the pixel equations to macro catchment scale equations; (3) an “empirical lumping” of new dominant processes detectable in the distributed model, but generally not described by the catchment scale equations. The process lumping is based upon the integration of the point equations over a pixel, while the parameter lumping is illustrated by means of a Monte Carlo simulation over a simple plane; finally the geo-morphological lumping and the empirical lumping are demonstrated on a set of sub-catchments of the Reno river in Italy..

Introduction

Presently, physically based distributed models are recognized to be the most appropriate tools for simulating and investigating hydrological processes. Nevertheless the power of synthesis of the lumped models makes them potentially very attractive. The interest in lumped models not only lies in the practical aim of using simpler models but even more in the theoretical objective of finding the “dominant processes”, namely the essential hydrological features, to be preserved in the lumping process.

This paper aims at showing that, unfortunately, the physical properties of the basic processes can only be retained at finer spatial scales, while, due to the inherent topological non-linearity, physically based lumped models can only be derived through an averaging process conditional upon a correct representation of additional phenomena (such as the soil filling and depletion hysteresis and the exfiltration at the end of a rainfall event), which can be obtained via a distributed modelling approach.

The proposed methodology

The aim of this work is to show that a lumped formulation of the hydrologic governing equations at the catchment scale is possible also preserving the physical meaning of the phenomena represented and of the model parameters. One of the key concepts is that formulation can not be directly derived at the lumped form, i.e. at the catchment scale. Instead this approach is, for instance, an important limit of the TOPMODEL-like formulations. As a matter of fact for those models, thank to the steady state assumption, the mass and momentum conservative equations can be solved directly at the catchment scale. But the price of this operation is the loss of the physical meaning of the model parameters and with that the consequent weakness of the physical representation of the phenomena. On the contrary, in this work the lumped formulation of the governing equations at the catchment scale is obtained by a sequence of equation integration from the point scale to the finite scale (the elementary grid cell), from the finite element to the catchment. This approach is showed to be capable of preserving the physical meaning framework of the hydrologic representation at different time-space scales of practise interest.

Another innovative aspect of the methodology is the proposed derivation of the contributing saturated area based on aggregated variables such as the total soil water content. As a mater of fact the governing equations once solved at the catchment scale – although derived by a correct procedure – do not allow, by definition, to estimate any variable which depends on the specific spatial configuration of the catchment state since it is described by means of average values. This is the case of the extension of the saturated area which plays indeed an important role on the runoff mechanism.

Model Lumping

A detailed description of the non-linear reservoir formulation for the lumped model at the catchment scale is provided by Liu and Todini 2002, Todini and Ciarapica 2001. the authors recall here only the final results of that derivation.

The first assumption is the the authors assume that the overall behaviour of the basin does not depend on the position of the individual slopes, but essentially on the global distribution of the slopes themselves; this assumption allows the lumped representation of the topographic trend of the basin. In the transition to the lumped structure of the model, this trend is no longer described according to its actual distribution, but rather according to a general distribution function.

The second assumption is that during the transition phase the variation over time in the water content is not ignored but assumed to be constant in space. This is the fundamental assumption on which the development of the model is based and whose validation forms the starting point of this study.

Our interest was to test by means of numerical experiments, the sensitivity of the solutions to the grid scale of the model. For this purpose the authors first distinguish the errors involved by the numerical scheme from that implied by the parameters averaging, then the authors looked at the simultaneous effects.

The experiments showed that the model, structured according to the proposed integration method, produces acceptable results upon a grid scale of the order of the kilometer. Beyond that scale, the greater divergences from the correct solution (i.e. considering the parameters spatial variability and using a very fine grid resolution) are found on the subsurface flow showing an evident dependency of the solution on the spatial distribution of the parameters within the grid cell. However, within the scale range, the good phenomena representation is achieved keeping the physical meaning of the model parameters since they are mere average then the real ones.

It seems then in order to preserve the essential feature of the rainfall-runoff process from the point scale to the grid cell scale and larger scales, representations that integrate the point equations may correctly reproduce phenomena such as the soil moisture balance and the formation of surface runoff. Therefore a correct integration of the differential equations from the point to the finite dimension of a grid cell, and from the grid cell to larger scales, can actually generate relatively scale independent models, which preserve the physical meaning (although as averages) of the model parameters.

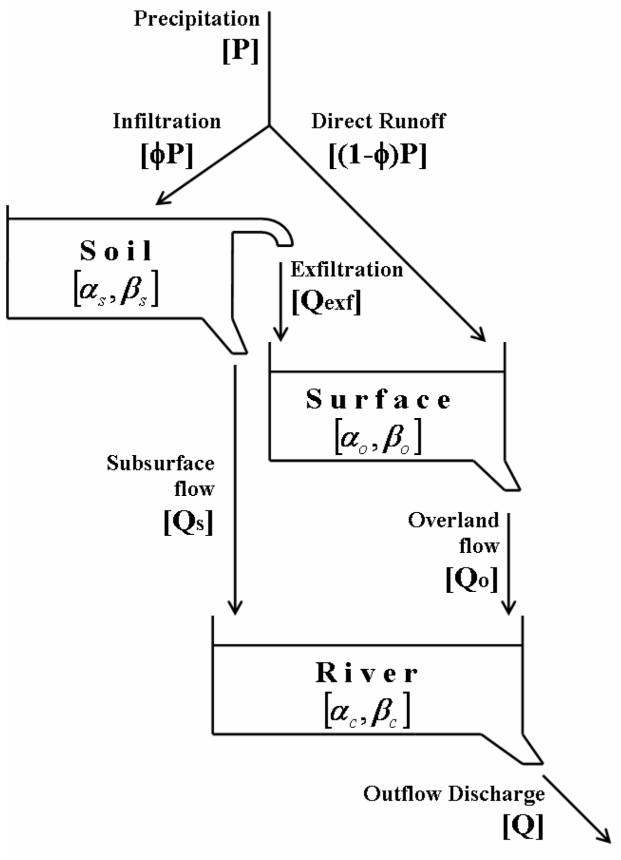

Figure 1 Conceptual scheme of the TOPKAPI lumped model.

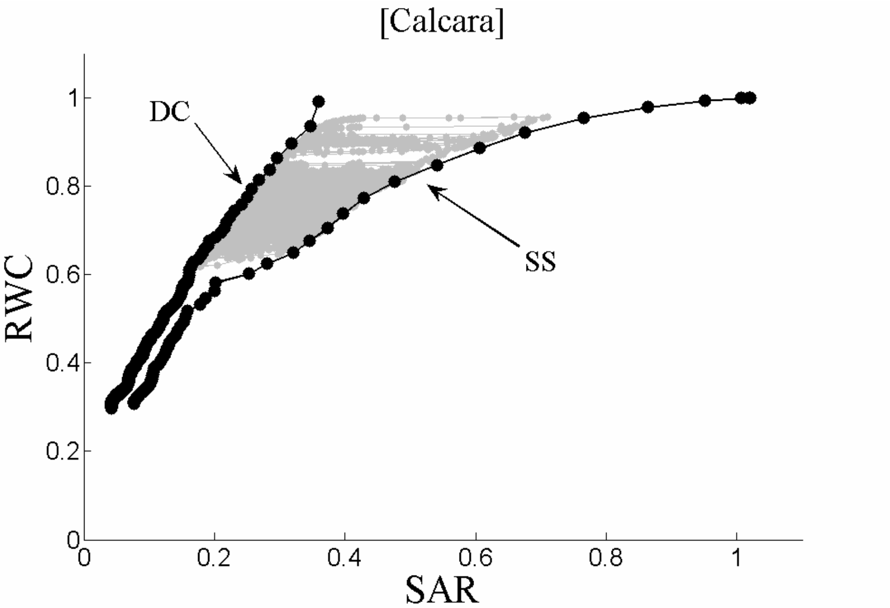

Figure 2 Saturated Area Ratio (SAR) vs. Relative Water Content (RWC) for a 5 year simulation period of the Reno catchment at the Calcara river section (grey dots). Steady State curve (SS) obtained as a set of SAR-RWC values for different simulations relative to precipitation events of different intensity and after the equilibrium state has been reached. Draying Curve is the SAR and RWC values for the only drying down transition phase.

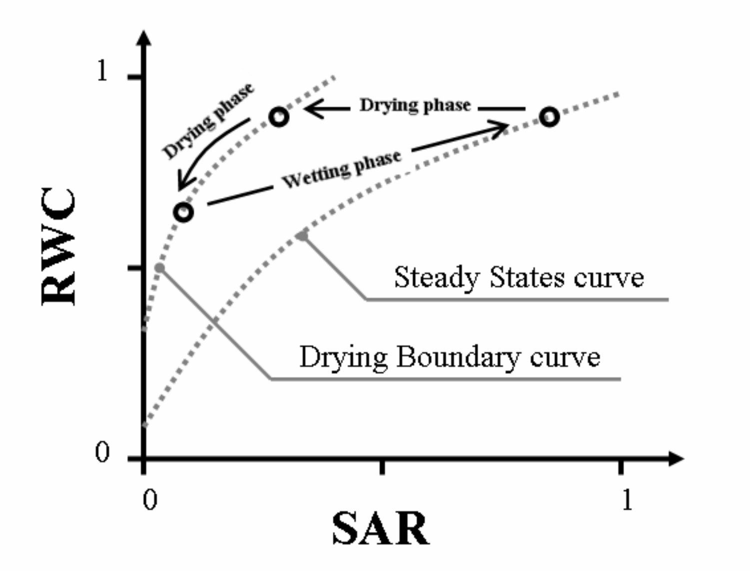

Figure 3 Conceptual scheme of the saturated area extension dynamics for the wetting up and drying down phases on the SAR-RWC plan.

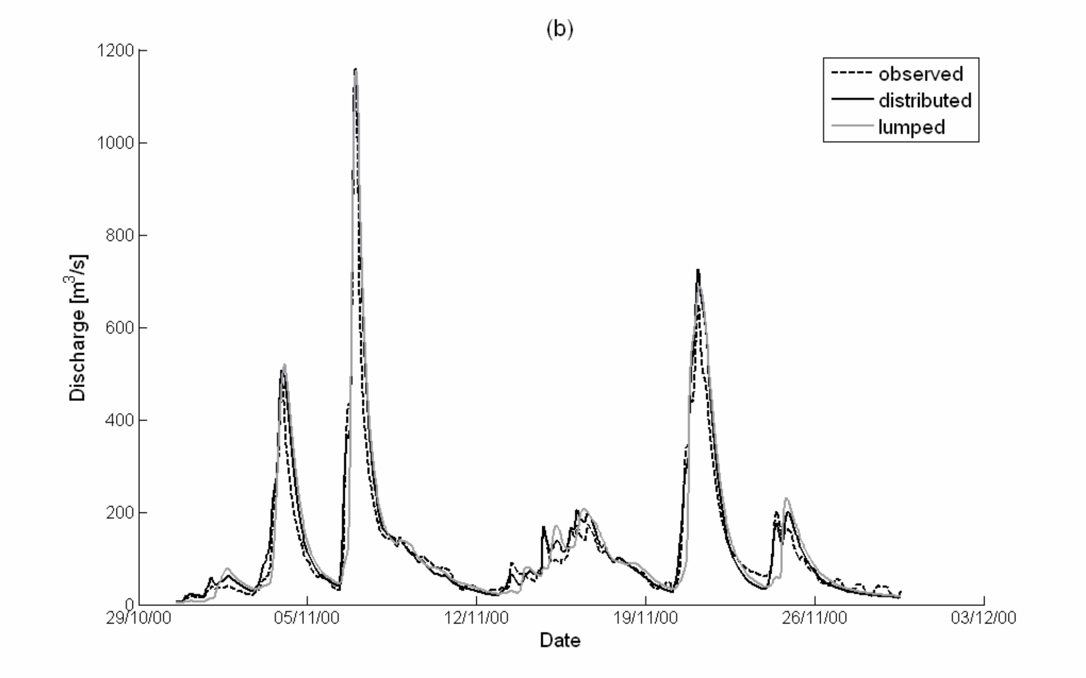

Figure 4 Hydrographs for the Reno catchment at the Casalecchio river section as simulated by the distributed and lumped TOPKAPI model compared with the observed. (a) a 3-year timeseries, (b) the major flood event, November 2000.

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